TKN/KS/16/5884

Bachelor of Science (B.Sc.) Semester—V
(C.B.S.) Examination
MATHEMATICS
Paper—II
(M_{10}-Metric Space, Complex Integration and Algebra)

Time—Three Hours]

[Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.
(2) All questions carry equal marks.
(3) Question No. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Let A be a countable set, and let B_n be the set of all n-tubles $(a_1, \ldots, a_n)$, where $a_k \in A (k = 1, \ldots, n)$, and the elements $a_1, \ldots, a_n$ need not be distinct. Then prove that B_n is countable. 6

(B) For $x \in R^1$ and $y \in R^1$, define $d(x, y) = (x - y)^2$. Determine, whether d is a metric or not. 6

OR

(C) If p is a limit point of a set E, then prove that every neighbourhood of p contains infinitely many points of E. 6

UNIT—III

3. (A) If R is a ring, then for all $a, b \in R$ prove that :

(i) $a\,o = o = o\,a$

(ii) $a(-b) = -(ab) = (-a)b$ and $(-a)(-b) = ab$.

6

(B) If ϕ is a homomorphism of R into R', then prove that :

(i) $\phi(o) = o'$ and

(ii) $\phi(-a) = -\phi(a)$

for every $a \in R$, where R and R' are the rings and o, o' are additive identities of R, R' respectively.

6

OR

(C) Let R, R' be rings and ϕ a homomorphism of R onto R' with kernel U. Then prove that R' is isomorphic to R/U. 6

(D) If U is an ideal of R, let $[R : U] = \{x \in R / rx \in U$ for every $r \in R\}$, prove that $[R : U]$ is an ideal of R and that it contains U. 6

UNIT—IV

4. (A) Calculate $\int_C \frac{z\,dz}{(9-z^2)(z+i)}$ by using Cauchy integral formula, where C is the circle $|\tau| = 2$ described in positive sense. 6

(B) Find the value of the integral $\int_0^{1+i} (x - y + ix^2)\,dz$ along the real axis from z = 0 to z = 1 and then along a line parallel to imaginary axis from z = 1 to z = 1+i. 6

OR

(C) If f(z) is analytic within and on a closed contour C, except at a finite number of poles $z_1, z_2, z_3, \ldots, z_n$ within C, then prove that :

$$\int_C f(z)\,dz = 2\pi i \sum_{r=1}^{n} \text{Res}\,(z = z_r).$$ 6

(D) Prove that $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta}\,d\theta = \frac{\pi}{6}$. 6

5. (A) Let $A = \{x \in R / 0 < x \le 1\}$ and $E_x = \{y \in R / 0 < y < x, x \in A\}$. If $E_x \subset E_z$, then prove that $0 < x \le z \le 1$. 1½

(B) If X is a metric space and $E \subset X$, then prove that $E = \overline{E}$ if and only if E is closed, where $\overline{E}$ = closure of E. 1½

(C) If $\{K_n\}$ is a sequence of nonempty compact sets such that $K_n \supset K_{n+1}$ (n = 1, 2, 3, ...), then prove that $\bigcap_1^{\infty} K_n$ is not empty. 1½

(D) Prove that a set E is open if and only if its complement is closed. 6

UNIT—II

2. (A) Suppose $K \subset Y \subset X$. Then prove that K is compact relative to X if and only if K is compact relative to Y. 6

(B) If $\{K_\alpha\}$ is a collection of compact subsets of a metric space X such that the intersection of every finite subcollection of $\{K_\alpha\}$ is nonempty, then prove that $\bigcap_\alpha K_\alpha$ is nonempty. 6

OR

(C) Let X be a complete metricspace and let $\{G_n\}$ be a decreasing sequence of nonempty closed subsets of X such that $\lim d(G_n) = 0$. Then prove that $\bigcap_{n=1}^{\infty} G_n$ contains exactly one point. 6

(D) Let E be a subset of the real line R^1. If $x, y \in E$ and $x < z < y \Rightarrow z \in E$, then prove that E is connected. 6

(D) Prove that every bounded infinite subset of R^k has a limit point in R^k. 1½

(E) Let R be a ring. If $a, b, c, d \in R$, then evaluate $(a + b)(c + d)$. 1½

(F) If R is a ring with unit element 1 and ϕ is a homomorphism of R onto a ring R', prove that $\phi(1)$ is the unit element of R'. 1½

(G) Show that $\int_C |z| dz = -2$, where C is the upper half part of circle $|z| = 1$. 1½

(H) What kind of singularity have the given function $\sin z - \cos z$ at $z = \infty$? 1½